Using Inductive Reasoning to Make Conjectures

Objective

Use inductive reasoning to identify patterns and make conjectures.

Find counterexamples to disprove conjectures.

Lesson Presentation

Lesson Review
**Objectives**

Use inductive reasoning to identify patterns and make conjectures.

Find counterexamples to disprove conjectures.
Example 1A: Identifying a Pattern

Find the next item in the pattern.

January, March, May, ...

*Alternating months of the year make up the pattern.*

The next month is July.

Example 1B: Identifying a Pattern

Find the next item in the pattern.

7, 14, 21, 28, ...

*Multiples of 7 make up the pattern.*

The next multiple is 35.
In this pattern, the figure rotates 90° counter-clockwise each time.

The next figure is △.
Find the next item in the pattern 0.4, 0.04, 0.004, ...

When reading the pattern from left to right, the next item in the pattern has one more zero after the decimal point.

The next item would have 3 zeros after the decimal point, or 0.0004.
When several examples form a pattern and you assume the pattern will continue, you are applying inductive reasoning. **Inductive reasoning** is the process of reasoning that a rule or statement is true because specific cases are true. You may use inductive reasoning to draw a conclusion from a pattern. A statement you believe to be true based on inductive reasoning is called a **conjecture**.
Example 2A: Making a Conjecture

Complete the conjecture.

The sum of two positive numbers is __?__.

*List some examples and look for a pattern.*

\[
1 + 1 = 2 \quad 3.14 + 0.01 = 3.15 \\
3,900 + 1,000,017 = 1,003,917
\]

The sum of two positive numbers is **positive**.
Complete the conjecture.

The product of two odd numbers is ___.

List some examples and look for a pattern.

\[ 1 \times 1 = 1 \quad 3 \times 3 = 9 \quad 5 \times 7 = 35 \]

The product of two odd numbers is odd.
Example 3: Biology Application

The cloud of water leaving a whale’s blowhole when it exhales is called its blow. A biologist observed blue-whale blows of 25 ft, 29 ft, 27 ft, and 24 ft. Another biologist recorded humpback-whale blows of 8 ft, 7 ft, 8 ft, and 9 ft. Make a conjecture based on the data.

<table>
<thead>
<tr>
<th>Heights of Whale Blows</th>
<th>25</th>
<th>29</th>
<th>27</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of Blue-whale Blows</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Height of Humpback-whale Blows</td>
<td>8</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

The highest blue-whale blow (24 ft) is about three times higher than the greatest humpback-whale height (9 ft). The blue whale’s blow is greater than a humpback whale’s blow.
Check for understanding

Make a conjecture about the lengths of male and female whales based on the data.

<table>
<thead>
<tr>
<th>Average Whale Lengths</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Female (ft)</td>
</tr>
<tr>
<td>Length of Male (ft)</td>
</tr>
</tbody>
</table>

In 5 of the 6 pairs of numbers above the female is longer.

Female whales are longer than male whales.
To show that a conjecture is always true, you must prove it.

To show that a conjecture is false, you have to find only one example in which the conjecture is not true. This case is called a **counterexample**.

A counterexample can be a drawing, a statement, or a number.
**Inductive Reasoning**

1. Look for a pattern.

2. Make a conjecture.

3. Prove the conjecture or find a counterexample.
Example 4A: Finding a Counterexample

Show that the conjecture is false by finding a counterexample.
For every integer \( n \), \( n^3 \) is positive.

Pick integers and substitute them into the expression to see if the conjecture holds.

Let \( n = 1 \). Since \( n^3 = 1 \) and \( 1 > 0 \), the conjecture holds.

Let \( n = -3 \). Since \( n^3 = -27 \) and \( -27 \leq 0 \), the conjecture is false.

\( n = -3 \) is a counterexample.
Example 4B: Finding a Counterexample

Show that the conjecture is false by finding a counterexample.

Two complementary angles are not congruent.

\[ 45° + 45° = 90° \]

*If the two congruent angles both measure 45°, the conjecture is false.*
Show that the conjecture is false by finding a counterexample.
For any real number $x$, $x^2 \geq x$.

Let $x = \frac{1}{2}$.

Since $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$, $\frac{1}{4} \neq \frac{1}{2}$.

The conjecture is false.
Example 4C: Finding a Counterexample

Show that the conjecture is false by finding a counterexample.

The monthly high temperature in Abilene is never below 90°F for two months in a row.

<table>
<thead>
<tr>
<th>Monthly High Temperatures (°F) in Abilene, Texas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
</tr>
<tr>
<td>88</td>
</tr>
</tbody>
</table>

The monthly high temperatures in January and February were 88°F and 89°F, so the conjecture is false.
Show that the conjecture is false by finding a counterexample.
The radius of every planet in the solar system is less than 50,000 km.

<table>
<thead>
<tr>
<th>Planets’ Diameters (km)</th>
<th>Mercury</th>
<th>Venus</th>
<th>Earth</th>
<th>Mars</th>
<th>Jupiter</th>
<th>Saturn</th>
<th>Uranus</th>
<th>Neptune</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>4880</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Venus</td>
<td>12,100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Earth</td>
<td>12,800</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mars</td>
<td>6790</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jupiter</td>
<td>143,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Saturn</td>
<td>121,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uranus</td>
<td>51,100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neptune</td>
<td>49,500</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since the radius is half the diameter, the radius of Jupiter is 71,500 km and the radius of Saturn is 60,500 km. The conjecture is false.
Lesson Review

Find the next item in each pattern.

1. 0.7, 0.07, 0.007, ...   2. 0.0007

Determine if each conjecture is true. If false, give a counterexample.

3. The quotient of two negative numbers is a positive number.  \textbf{true}

4. Every prime number is odd.  \textbf{false}; 2

5. Two supplementary angles are not congruent.  \textbf{false}; 90^\circ \text{ and } 90^\circ

6. The square of an odd integer is odd.  \textbf{true}